ON THE INVERSION OF POTENTIAL TYPE OPERATORS WITH KERNELS HAVING SINGULARITIES ON A SPHERE *

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Abstract

The inversion problem for operators with symbols $a(|\xi|)|\xi|^{-\alpha}e^{i|\xi|}$ is considered in the framework of L_p spaces. It is assumed that $1 \leq p \leq \infty$, $\operatorname{Re}\alpha \geq 0$, $a(|\xi|)$ is a $L_1 - L_1$ - multiplier and satisfies some additional properties, depending on the case, that we consider. We treat here the particular important non elliptic case, in which $a(|\xi|) \neq 0$, $\xi \neq 0$, but in which $a(|\xi|)$ may degenerate at the origin and at infinity. We also consider the general case of non-ellipticity, when $a(|\xi|)$ degenerates on an arbitrary set of zero measure in $\mathbb{R}^n$. Additionally, we give information on the action of the operators A_a^α on L_p spaces.

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1. Introduction

Within the framework of the spaces $L_p \equiv L_p(\mathbb{R}^n)$ in this paper we consider the inversion problem for the operators A_a^α with symbols

$$m_{a,\alpha}(\xi) = a(|\xi|)|\xi|^{-\alpha}e^{i|\xi|}, \quad \operatorname{Re}\alpha \geq 0.$$

Here $a(|x|) \in M_1^1$, the class of $L_1 - L_1$ multipliers, and satisfies some additional properties in each specific case (see below). For $\frac{n+1}{2} < \operatorname{Re}\alpha < n$ they are realized as potential-type convolution operators having the kernels

$$\Omega_{\alpha,a}(|x|) = (2\pi)^{-\frac{n}{2}}|x|^{\alpha-n}\int_0^\infty t^{\frac{n}{2}-\alpha}a\left(\frac{t}{|x|}\right)e^{\frac{it}{|x|}}\mathcal{J}_{\frac{n}{2}-1}(t)dt, \tag{1.1}$$

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